Nema opisa.

ZADATAK 7.2.1
Dan je paralelogram $ABCD$. Neka je točka S sjecište njegovih dijagonala. Izračunaj:

1) $\overrightarrow{AD}+\overrightarrow{CD}$

2) $\overrightarrow{AS}+\overrightarrow{BS}$

3) $\overrightarrow{AD}+\overrightarrow{CB}$

4) $\overrightarrow{AB}+\overrightarrow{SD}$

5) $\overrightarrow{AB}+\overrightarrow{BS}$

6) $\overrightarrow{BS}+\overrightarrow{CS}$.

RJEŠENJE
1) $\overrightarrow{AD}+\overrightarrow{CD}=\overrightarrow{BC}+\overrightarrow{CD}=\overrightarrow{BD}$

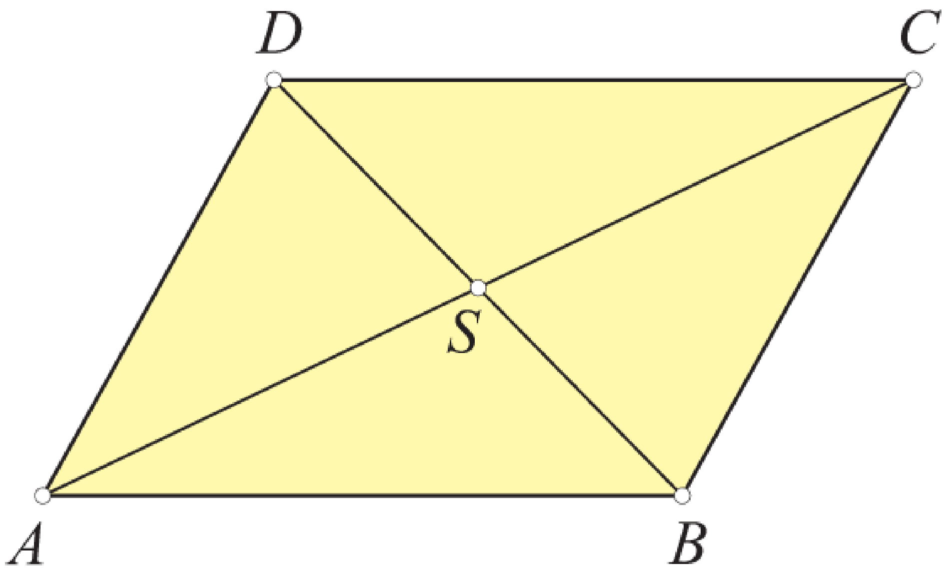
2) $\overrightarrow{AS}+\overrightarrow{BS}=\overrightarrow{AS}+\overrightarrow{SD}=\overrightarrow{AD}$

3) $\overrightarrow{AD}+\overrightarrow{CB}=\overrightarrow{AD}+\overrightarrow{DA}=\overrightarrow{AD}-\overrightarrow{AD}=\vec{0}$

4) $\overrightarrow{AB}+\overrightarrow{SD}=\overrightarrow{AB}+\overrightarrow{BS}=\overrightarrow{AS}$

5) $\overrightarrow{AB}+\overrightarrow{BS}=\overrightarrow{AS}$

6) $\overrightarrow{BS}+\overrightarrow{CS}=\overrightarrow{BS}+\overrightarrow{SA}=\overrightarrow{BA}$.



ZADATAK 7.2.3

Neka je $ABCDEF$ pravilni šesterokut i neka je S sjecište njegovih dijagonala. Izračunaj:

1) $\overrightarrow{AB}+\overrightarrow{EF}$

2) $\overrightarrow{AB}+\overrightarrow{SD}$

3) $\overrightarrow{BC}+\overrightarrow{ES}$

4) $\overrightarrow{CS}+\overrightarrow{EF}$

5) $\overrightarrow{DE}+\overrightarrow{SC}$

6) $\overrightarrow{CF}+\overrightarrow{AS}$.

RJEŠENJE

1) $\overrightarrow{AB}+\overrightarrow{EF}=\overrightarrow{AB}+\overrightarrow{SA}=\overrightarrow{SA}+\overrightarrow{AB}=\overrightarrow{SB}$

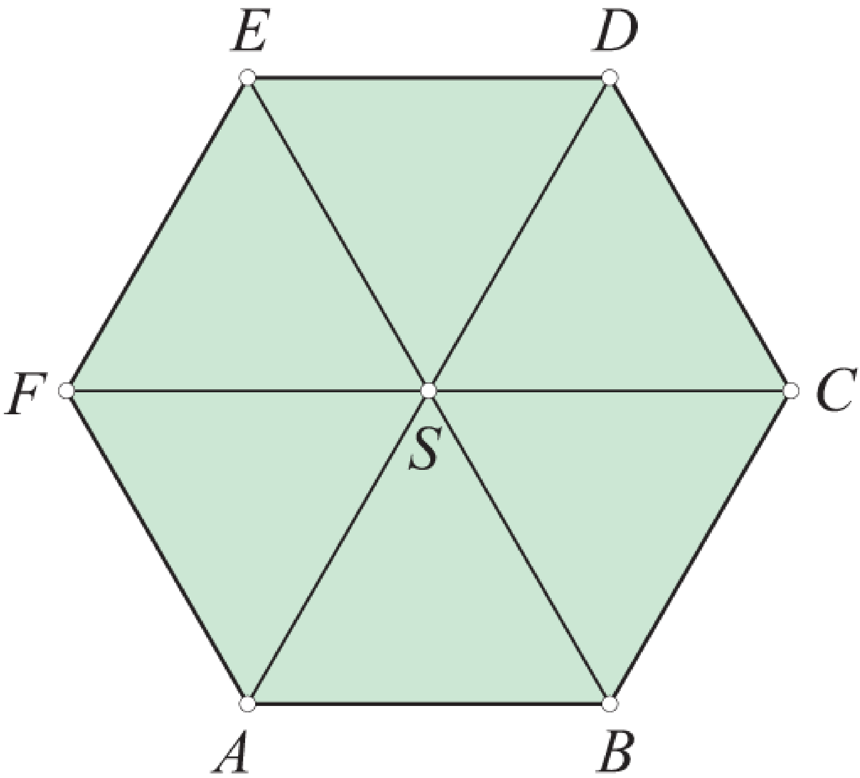
2) $\overrightarrow{AB}+\overrightarrow{SD}=\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$

3) $\overrightarrow{BC}+\overrightarrow{ES}=\overrightarrow{ES}+\overrightarrow{BC}=\overrightarrow{ES}+\overrightarrow{SD}=\overrightarrow{ED}$

4) $\overrightarrow{CS}+\overrightarrow{EF}=\overrightarrow{CS}+\overrightarrow{SA}=\overrightarrow{CA}$

5) $\overrightarrow{DE}+\overrightarrow{SC}=\overrightarrow{DE}+\overrightarrow{ED}=\overrightarrow{DE}-\overrightarrow{DE}=\vec{0}$

6) $\overrightarrow{CF}+\overrightarrow{AS}=\overrightarrow{CF}+\overrightarrow{FE}=\overrightarrow{CE}$.



ZADATAK 7.2.4

Točka S sjecište je dijagonala pravilnog šesterokuta $ABCDEF$. Izračunaj:

1) $\overrightarrow{AB}+\overrightarrow{SD}+\overrightarrow{SF}$

2) $\overrightarrow{AB}+\overrightarrow{CD}+\overrightarrow{EF}$

3) $\overrightarrow{AB}+\overrightarrow{AS}+\overrightarrow{AF}$

4) $\overrightarrow{SB}+\overrightarrow{SD}+\overrightarrow{SF}$.

RJEŠENJE

1) $\overrightarrow{AB}+\overrightarrow{SD}+\overrightarrow{SF}=(\overrightarrow{AB}+\overrightarrow{AS})+\overrightarrow{SF}$

$=(\text{po pravilu paralelograma})=\overrightarrow{AC}+\overrightarrow{SF}$

$=\overrightarrow{AC}+\overrightarrow{CS}=\overrightarrow{AS}$

2) $\overrightarrow{AB}+\overrightarrow{CD}+\overrightarrow{EF}=\overrightarrow{AB}+\overrightarrow{BS}+\overrightarrow{EF}$

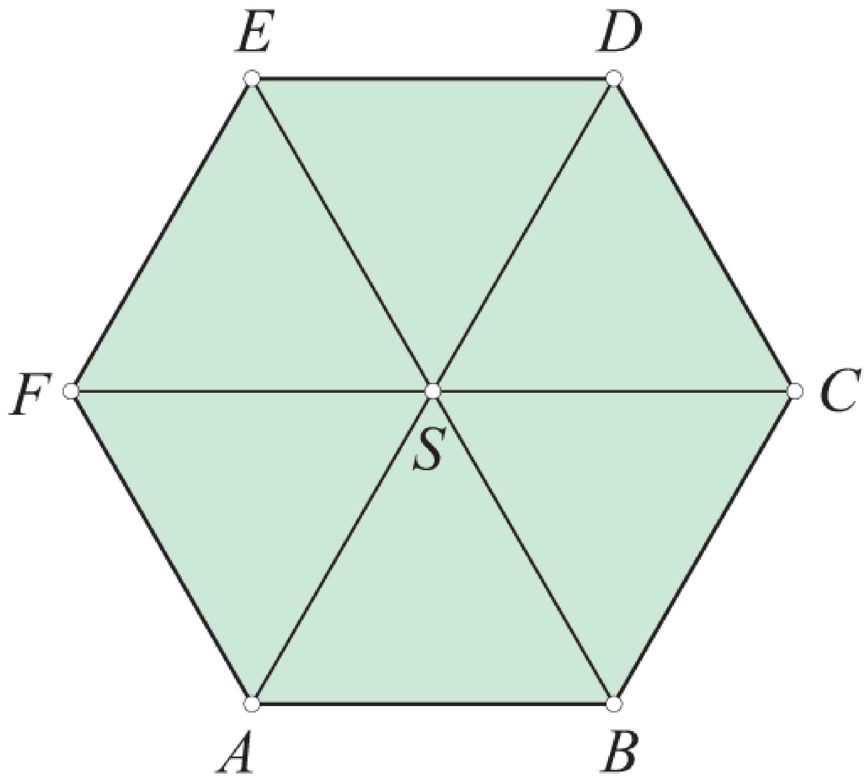
$=\overrightarrow{AS}+(-\overrightarrow{AS})=\vec{0}$

3) $\overrightarrow{AB}+\overrightarrow{AS}+\overrightarrow{AF}=(\overrightarrow{AB}+\overrightarrow{AF})+\overrightarrow{AS}=\overrightarrow{AS}+\overrightarrow{AS}$

$=2\overrightarrow{AS}=\overrightarrow{AD}$

4) $\overrightarrow{SB}+\overrightarrow{SD}+\overrightarrow{SF}=(\overrightarrow{SB}+\overrightarrow{SF})+\overrightarrow{SD}=\overrightarrow{SA}+\overrightarrow{SD}$.

$=\overrightarrow{SA}+(-\overrightarrow{SA})=\vec{0}$



ZADATAK 7.2.6
Odredi zbroj vektora:

1) $\overrightarrow{AC}+\overrightarrow{DB}+\overrightarrow{CD}+\overrightarrow{BA}$

2) $\overrightarrow{AB}+\overrightarrow{CD}+\overrightarrow{BC}+\overrightarrow{DE}$

3) $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}$

4) $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{DA}$

5) $\overrightarrow{AB}+\overrightarrow{BD}+\overrightarrow{BC}+\overrightarrow{DB}$.

RJEŠENJE

1) $\overrightarrow{AC}+\overrightarrow{DB}+\overrightarrow{CD}+\overrightarrow{BA}=(\overrightarrow{AC}+\overrightarrow{CD})+\overrightarrow{DB}+\overrightarrow{BA}$
$\quad=\overrightarrow{AD}+\overrightarrow{DB}+\overrightarrow{BA}=\overrightarrow{AB}+\overrightarrow{BA}=\vec{0}$

2) $\overrightarrow{AB}+\overrightarrow{CD}+\overrightarrow{BC}+\overrightarrow{DE}=(\overrightarrow{AB}+\overrightarrow{BC})+(\overrightarrow{CD}+\overrightarrow{DE})$
$\quad=\overrightarrow{AC}+\overrightarrow{CE}=\overrightarrow{AE}$

3) $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=\overrightarrow{AC}+\overrightarrow{CA}=\vec{0}$

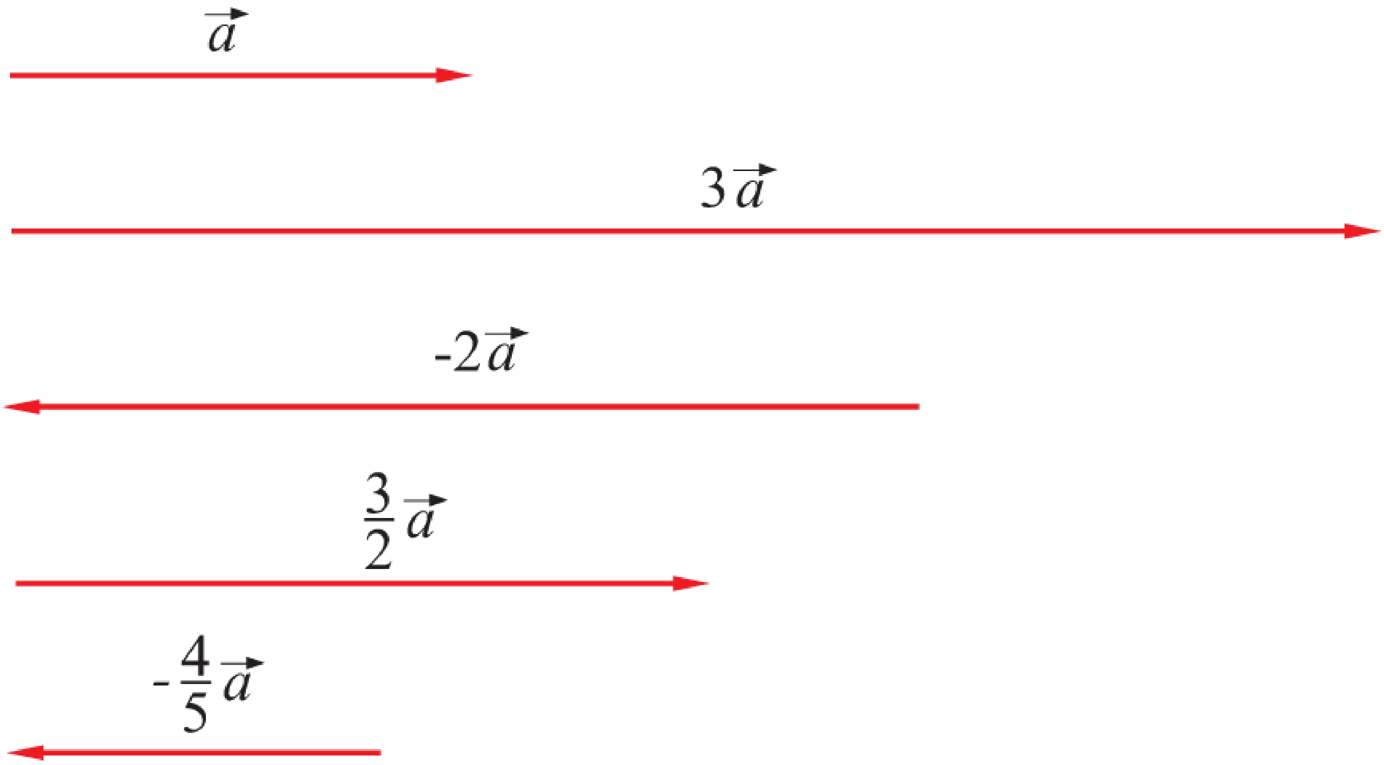
4) $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{DA}=\overrightarrow{AC}+\overrightarrow{DA}=\overrightarrow{DA}+\overrightarrow{AC}=\overrightarrow{DC}$

5) $\overrightarrow{AB}+\overrightarrow{BD}+\overrightarrow{BC}+\overrightarrow{DB}=\overrightarrow{AB}+\overrightarrow{BD}+\overrightarrow{DB}+\overrightarrow{BC}$.
$\quad=\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$

ZADATAK 7.3.1
Nacrtaj neki vektor $\vec{a}$ pa konstruiraj vektore

$$3\vec{a},\quad -2\vec{a},\quad \frac{3}{2}\vec{a},\quad -\frac{4}{5}\vec{a}.$$

RJEŠENJE

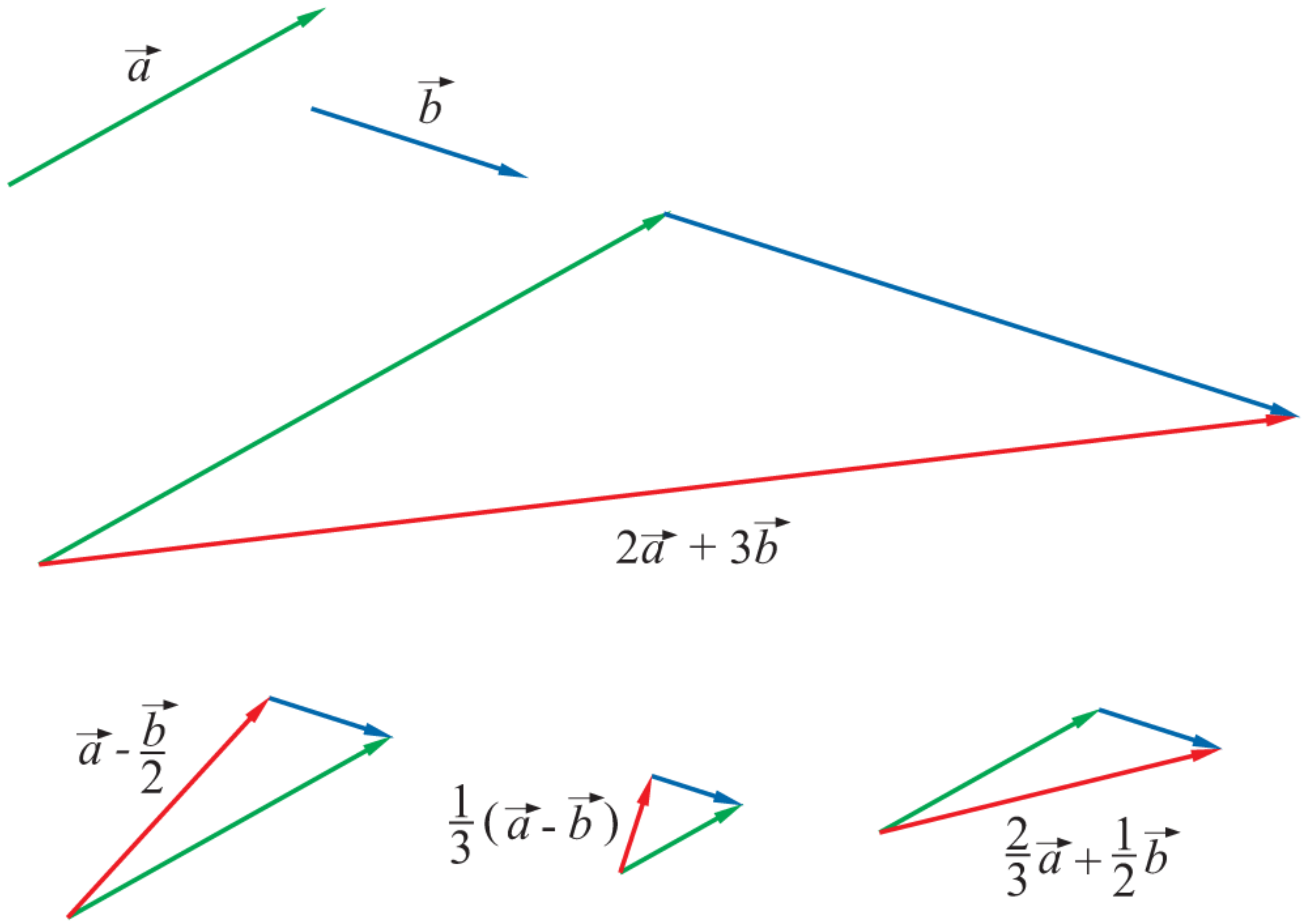


ZADATAK 7.3.2

Nacrtaj dva vektora $\vec{a}$ i $\vec{b}$ pa konstruiraj vektore

$$2\vec{a}+3\vec{b}, \quad \vec{a}-\frac{\vec{b}}{2}, \quad \frac{1}{3}(\vec{a}-\vec{b}), \quad \frac{2}{3}\vec{a}+\frac{1}{2}\vec{b}.$$

RJEŠENJE



ZADATAK 7.3.3

Za koje vrijednosti broja k su vektori $\vec{a}$ i $k \cdot \vec{a}$

1) jednaki

2) suprotni

3) iste orijentacije?

RJEŠENJE

1) $k=1$

2) $k=-1$

3) $k>0$.

ZADATAK 7.3.4

Dan je četverokut $ABCD$. Kakav je to četverokut ako su vektori $\overrightarrow{AB}$ i $\overrightarrow{DC}$

1) jednaki

2) kolinearni?

RJEŠENJE

1) Četverokut je paralelogram

2) četverokut je trapez.

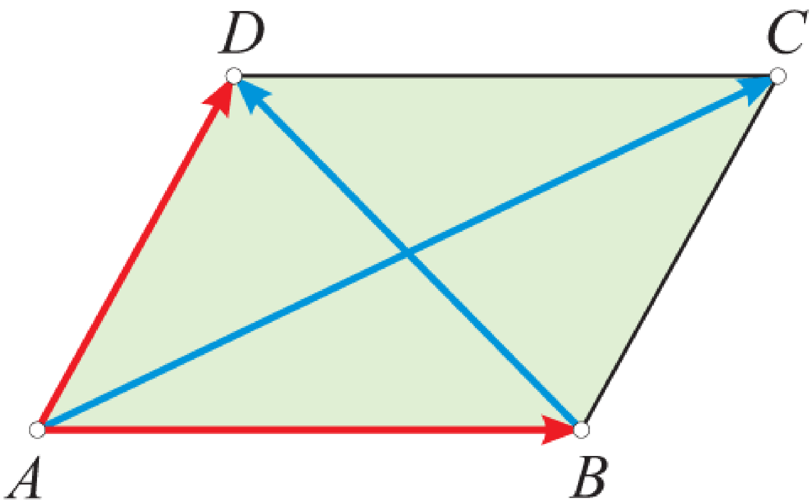
ZADATAK 7.4.2

Dan je paralelogram $ABCD$. Prikaži vektore $\overrightarrow{AD}$ i $\overrightarrow{AB}$ kao linearnu kombinaciju vektora $\overrightarrow{AC}$ i $\overrightarrow{BD}$.

RJEŠENJE

$\overrightarrow{AD}=\frac{1}{2}\overrightarrow{AC}+\frac{1}{2}\overrightarrow{BD}$

$\overrightarrow{AB}=\frac{1}{2}\overrightarrow{AC}-\frac{1}{2}\overrightarrow{BD}$.



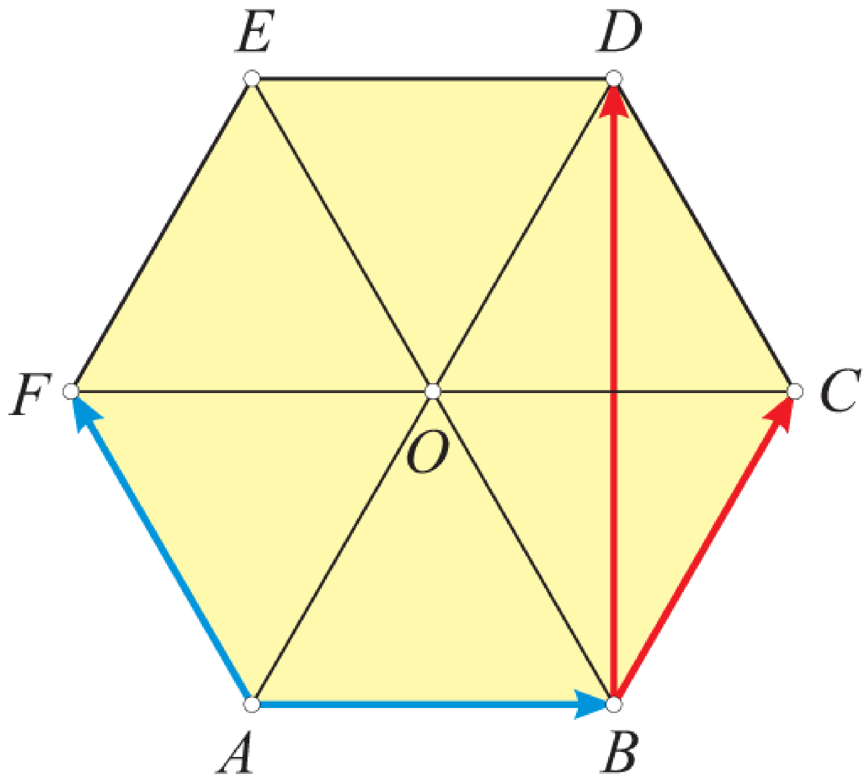
ZADATAK 7.4.6

Neka je $ABCDEF$ pravilni šesterokut. Izrazi vektore $\overrightarrow{BC}$ i $\overrightarrow{BD}$ kao linearnu kombinaciju vektora $\overrightarrow{AB}$ i $\overrightarrow{AF}$.

RJEŠENJE

$\overrightarrow{BC}=\overrightarrow{BO}+\overrightarrow{OC}=\overrightarrow{AF}+\overrightarrow{AB}$

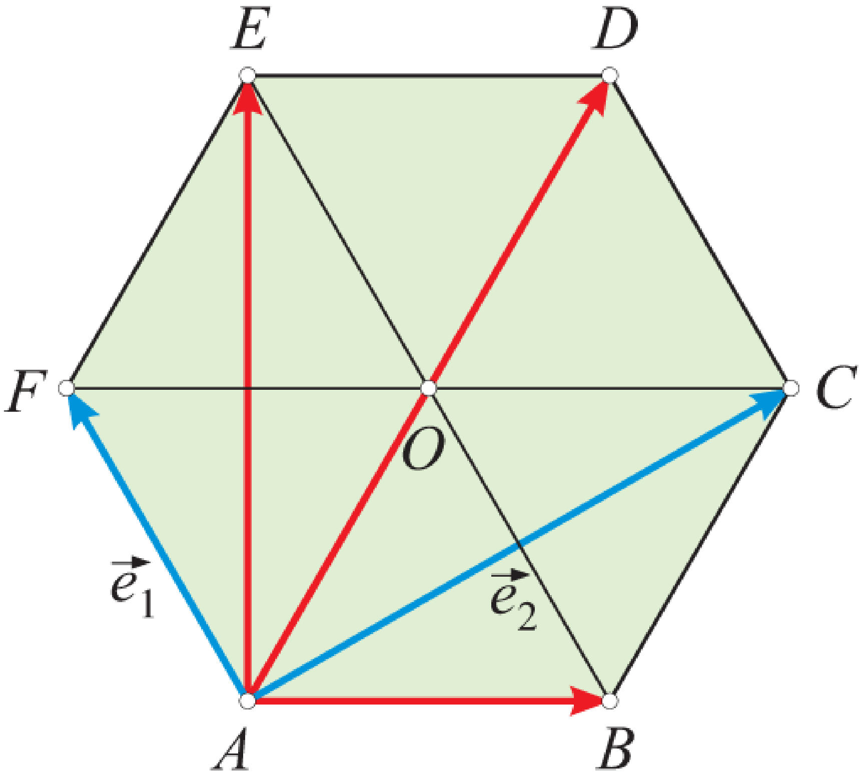
$\overrightarrow{BD}=\overrightarrow{BE}+\overrightarrow{ED}=2\overrightarrow{AF}+\overrightarrow{AB}$.



ZADATAK 7.4.7

ISPITNI Točke A, B, C, D, E i F vrhovi su pravilnog šesterokuta. Ako je $\overrightarrow{AF}=\vec{e}_1$, $\overrightarrow{AC}=\vec{e}_2$, prikaži vektore $\overrightarrow{AB}$, $\overrightarrow{AD}$ i $\overrightarrow{AE}$ kao linearnu kombinaciju vektora $\vec{e}_1$ i $\vec{e}_2$.

RJEŠENJE



$$\begin{aligned}\overrightarrow{AB}&=\vec{e}_2+\overrightarrow{CB}=\vec{e}_2+\overrightarrow{OA}\\&=\vec{e}_2+\overrightarrow{OF}+\overrightarrow{FA}=\vec{e}_2-\overrightarrow{AB}-\vec{e}_1\end{aligned}$$

$$\Longrightarrow\ 2\overrightarrow{AB}=\vec{e}_2-\vec{e}_1\ \Longrightarrow\ \overrightarrow{AB}=\frac{1}{2}\vec{e}_2-\frac{1}{2}\vec{e}_1$$

$$\overrightarrow{AD}=\overrightarrow{AC}+\overrightarrow{CD}=\vec{e}_1+\overrightarrow{AF}=\vec{e}_1+\vec{e}_2$$

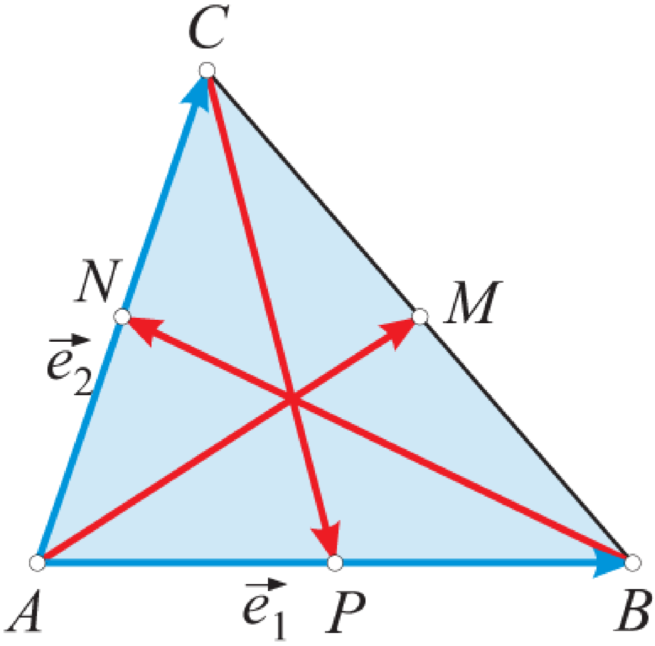
$$\overrightarrow{AE}=\overrightarrow{AB}+\overrightarrow{BE}=\frac{1}{2}\vec{e}_2-\frac{1}{2}\vec{e}_1+2\vec{e}_1=\frac{3}{2}\vec{e}_1+\frac{1}{2}\vec{e}_2\ .$$

ZADATAK 7.4.8

Neka su M, N i P polovišta stranica $= BC$, $= AC$ i $= AB$ trokuta ABC.

Prikaži vektore $\overrightarrow{AM}$, $\overrightarrow{BN}$ i $\overrightarrow{CP}$ kao linearne kombinacije vektora $\overrightarrow{AB} = \vec{e}_1$ i $\overrightarrow{AC} = \vec{e}_2$.

RJEŠENJE



$\overrightarrow{BC} = \vec{e}_1 - \vec{e}_1$

$$\overrightarrow{AM} = \overrightarrow{AB} + \overrightarrow{BM} = \vec{e}_1 + \frac{1}{2}\overrightarrow{CB} = \vec{e}_1 - \frac{1}{2}\overrightarrow{BC}$$

$$= \vec{e}_1 - \frac{1}{2}(\vec{e}_1 - \vec{e}_1) = \frac{1}{2}\vec{e}_1 + \frac{1}{2}\vec{e}_2$$

$$\overrightarrow{BN} = \overrightarrow{BA} + \overrightarrow{AN} = \frac{1}{2}\vec{e}_2 - \vec{e}_1$$

$$\overrightarrow{CP} = \overrightarrow{CA} + \overrightarrow{AP} = -\overrightarrow{AC} + \frac{1}{2}\overrightarrow{AB} = \frac{1}{2}\vec{e}_1 - \vec{e}_2\ .$$

ZADATAK 7.4.13

Ako su $\vec{m}$ i $\vec{n}$ nekolinearni vektori, odredi realni broj x tako da vektori $\vec{a}$ i $\vec{b}$, $\vec{a}=(x-1)\vec{m}+\vec{n}$ i $\vec{b}=3\vec{m}+(x+1)\vec{n}$ budu kolinearni.

RJEŠENJE

$\vec{m}$ i $\vec{n}$ su nekolinearni vektori.

$$\begin{aligned}&\vec{a}=(x-1)\vec{m}+\vec{n}\\&\vec{b}=3\vec{m}+(x+1)\vec{n}\\&\vec{a}=k\vec{b},\quad k\neq 0\\&(x-1)\vec{m}+\vec{n}=3k\vec{m}+k(x+1)\vec{n}\end{aligned}$$

Izjednačavanjem koeficijenata uz vektor m, odnosno n dobijemo sustav dviju jednadžbi s dvjema nepoznanicama:

$$\begin{aligned}&x-1=3k \implies k=\frac{x-1}{3}\\&1=k(x+1)\end{aligned}$$

Izrazimo li k iz prve jednadžbe i uvrstimo u drugu dobivamo:

$$\begin{aligned}&1=\frac{x-1}{3}(x+1)\\&3=x^2-1\\&x^2=4\\&x_1=2,\ \ x_2=-2\end{aligned}$$

Za $x=2$ je $\vec{b}=3\vec{a}$, a za $x=-2$ je $\vec{a}=-\vec{b}$.

ZADATAK 7.5.2
Dane su točke $A(-3,1)$, $B(-1,-2)$, $C(2,3)$.

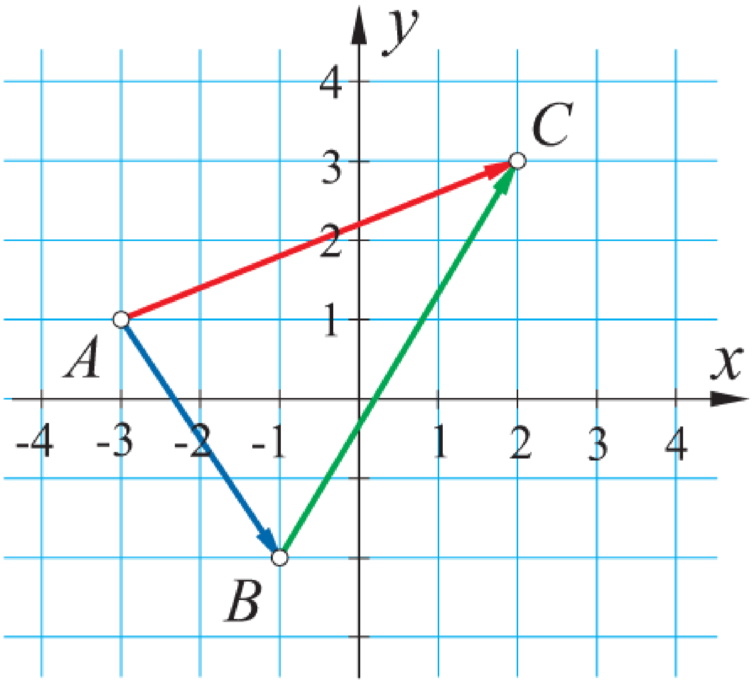
Odredi vektore $\overrightarrow{AB}$, $\overrightarrow{BC}$ i $\overrightarrow{AC}$.

Nakon toga ucrtaj vektore u kvadratnu koordinatnu mrežu i uvjeri se u točnost rješenja.

RJEŠENJE
$\overrightarrow{AB} = 2\vec{\imath} - 3\vec{\jmath}$

$\overrightarrow{BC} = 3\vec{\imath} + 5\vec{\jmath}$

$\overrightarrow{AC} = 5\vec{\imath} + 2\vec{\jmath}$



ZADATAK 7.5.4
Izračunaj duljine vektora:

$\vec{a} = 3\vec{\imath} + 4\vec{\jmath}$

$\vec{b} = \vec{\imath} - \vec{\jmath}$

$\vec{c} = -8\vec{\imath} + 6\vec{\jmath}$

$\vec{d} = 3\vec{\imath} - \vec{\jmath}$

$\vec{e} = 12\vec{\imath} - 5\vec{\jmath}$.

RJEŠENJE
$|\vec{a}| = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$

$|\vec{b}| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$|\vec{c}| = \sqrt{(-8)^2 + 6^2} = \sqrt{64+36} = \sqrt{100} = 10$

$|\vec{d}| = \sqrt{3^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$

$|\vec{e}| = \sqrt{12^2 + (-5)^2} = \sqrt{144+25} = \sqrt{169} = 13$.

ZADATAK 7.5.7

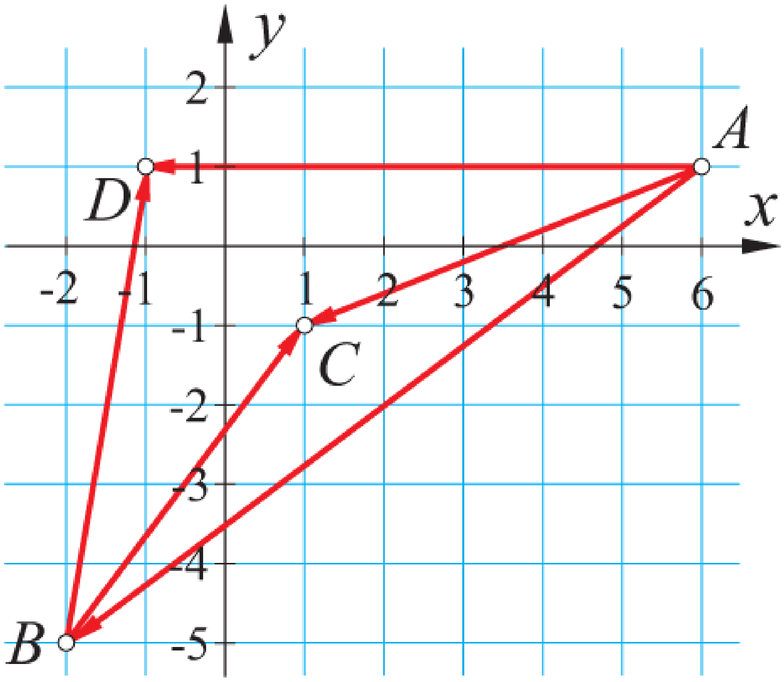
Dane su točke $A(6,1)$, $B(-2,-5)$, $C(1,-1)$, $D(-1,1)$.

Odredi vektore $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$, $\overrightarrow{BC}$, $\overrightarrow{BD}$ i izračunaj njihove duljine.

Provjeravaj rješenja ucrtavanjem točaka i vektora u koordinatni sustav.

RJEŠENJE

$A(6,1)$, $B(-2,-5)$, $C(1,-1)$, $D(-1,1)$



$\overrightarrow{AB} = -8\vec{\imath} - 6\vec{\jmath}$

$|\overrightarrow{AB}| = \sqrt{64+36} = 10$

$\overrightarrow{AC} = -5\vec{\imath} - 2\vec{\jmath}$

$|\overrightarrow{AC}| = \sqrt{25+4} = \sqrt{29}$

$\overrightarrow{AD} = -7\vec{\imath}$

$|\overrightarrow{AD}| = \sqrt{49} = 7$

$\overrightarrow{BC} = 3\vec{\imath} + 4\vec{\jmath}$

$|\overrightarrow{BC}| = \sqrt{9+16} = 5$

$\overrightarrow{BD} = \vec{\imath} + 6\vec{\jmath}$

$|\overrightarrow{BD}| = \sqrt{1+36} = \sqrt{37}$.

ZADATAK 7.5.8

ISPITNI Ako su $A(1,-1)$, $B(3,2)$ i $C(-2,3)$ tri uzastopna vrha paralelograma $ABCD$, odredi koordinate četvrtog vrha D.

RJEŠENJE

$$\begin{array}{l}
\underline{A(1,-1),\quad B(3,2),\quad C(-2,3),\quad D(x,y)}\\
\overrightarrow{AB}=\overrightarrow{DC}\quad \text{(paralelogram)}\\
(3-1)\vec{\imath}+(2+1)\vec{\jmath}=(-2-x)\vec{\imath}+(3-y)\vec{\jmath}\\
2\vec{\imath}+3\vec{\jmath}=(-2-x)\vec{\imath}+(3-y)\vec{\jmath}\\
2=-2-x\implies x=-4\\
3=3-y\implies y=0\\
\implies D(-4,0)
\end{array}$$

ZADATAK 7.5.9

Ako su $A(2,1)$, $B(-2,4)$ i $D(0,-3)$ tri vrha paralelograma $ABCD$, odredi koordinate vrha C.

RJEŠENJE

$$\begin{array}{l}
\underline{A(2,1),\quad B(-2,4),\quad C(x,y),\quad D(0,-3)}\\
\overrightarrow{AB}=\overrightarrow{DC}\quad \text{(paralelogram)}\\
(-2-2)\vec{\imath}+(4-1)\vec{\jmath}=(x-0)\vec{\imath}+(y+3)\vec{\jmath}\\
-4\vec{\imath}+3\vec{\jmath}=x\vec{\imath}+(y+3)\vec{\jmath}\\
-4=x\implies x=-4\\
3=y+3\implies y=0\\
\implies C(-4,0).
\end{array}$$

ZADATAK 7.5.10

Točke $A(0,3)$ i $B(2,2)$ dva su vrha paralelograma $ABCD$, a točka $S(3,4)$ sjecište je njegovih dijagonala.

Odredi koordinate vrhova C i D.

RJEŠENJE

$$\begin{array}{l}
\underline{A(0,3),\quad B(2,2),\quad S(3,4),\quad C(x_C,y_C),\quad D(x_D,y_D)}\\
C,\ D=?\\
\overrightarrow{AS}=\overrightarrow{SC}\\
(3-0)\vec{\imath}+(4-3)\vec{\jmath}=(x_C-3)\vec{\imath}+(y_C-4)\vec{\jmath}\\
3\vec{\imath}+1\vec{\jmath}=(x_C-3)\vec{\imath}+(y_C-4)\vec{\jmath}\\
3=x_C-3\implies x_C=6\\
1=y_C-4\implies y_C=5\\
\implies C(6,5)\\
\overrightarrow{BS}=\overrightarrow{SD}\\
(3-2)\vec{\imath}+(4-2)\vec{\jmath}=(x_D-3)\vec{\imath}+(y_D-4)\vec{\jmath}\\
\vec{\imath}+2\vec{\jmath}=(x_D-3)\vec{\imath}+(y_D-4)\vec{\jmath}\\
1=x_D-3\implies x_D=4\\
2=y_D-4\implies y_D=6\\
\implies D(4,6)
\end{array}$$

ZADATAK 7.5.11

Točke $B(1,-2)$ i $C(3,2)$ vrhovi su paralelograma $ABCD$, a točka $S\left(-\frac{1}{2},\frac{3}{2}\right)$ sjecište je njegovih dijagonala.

Odredi vrhove A i D ovog paralelograma.

RJEŠENJE

$$\begin{array}{l}
A(x_A,y_A),\quad B(1,-2),\quad S\left(-\frac{1}{2},\frac{3}{2}\right),\quad C(3,2),\quad D(x_D,y_D)\\
\hline
A,\ D=?\\
\overrightarrow{AS}=\overrightarrow{SC}\\
\left(-\frac{1}{2}-x_A\right)\vec{\imath}+\left(\frac{3}{2}-y_A\right)\vec{\jmath}=\left(3+\frac{1}{2}\right)\vec{\imath}+\left(2-\frac{3}{2}\right)\vec{\jmath}\\
\left(-\frac{1}{2}-x_A\right)\vec{\imath}+\left(\frac{3}{2}-y_A\right)\vec{\jmath}=\frac{7}{2}\vec{\imath}+\frac{1}{2}\vec{\jmath}\\
-\frac{1}{2}-x_A=\frac{7}{2}\implies x_A=-4\\
\frac{3}{2}-y_A=\frac{1}{2}\implies y_A=1\\
\implies A(-4,1)\\
\overrightarrow{BS}=\overrightarrow{SD}\\
\left(-\frac{1}{2}-1\right)\vec{\imath}+\left(\frac{3}{2}+2\right)\vec{\jmath}=\left(x_D+\frac{1}{2}\right)\vec{\imath}+\left(y_D-\frac{3}{2}\right)\vec{\jmath}\\
-\frac{3}{2}\vec{\imath}+\frac{7}{2}\vec{\jmath}=\left(x_D+\frac{1}{2}\right)\vec{\imath}+\left(y_D-\frac{3}{2}\right)\vec{\jmath}\\
-\frac{3}{2}=x_D+\frac{1}{2}\implies x_D=-2\\
\frac{7}{2}=y_D-\frac{3}{2}\implies y_D=5\\
\implies D(-2,5)
\end{array}$$

ZADATAK 7.5.12

Točke $A(-1,-1)$, $B(3,-2)$ i $C(5,2)$ tri su uzastopna vrha paralelograma $ABCD$.

Kolika je duljina dijagonale $=BD$?

RJEŠENJE

$$\begin{array}{l}
A(-1,-1),\quad B(3,-2),\quad C(5,2),\quad D(x_D,y_D)\\
\hline
|\overline{BD}|=?\\
\overrightarrow{AB}=\overrightarrow{DC}\\
(3+1)\vec{\imath}+(-2+1)\vec{\jmath}=(5-x_D)\vec{\imath}+(2-y_D)\vec{\jmath}\\
4\vec{\imath}-\vec{\jmath}=(5-x_D)\vec{\imath}+(2-y_D)\vec{\jmath}\\
5-x_D=4\implies x_D=1\\
2-y_D=-1\implies y_D=3\\
\implies D(1,3)\\
\overrightarrow{BD}=(1-3)\vec{\imath}+(3+2)\vec{\jmath}=-2\vec{\imath}+5\vec{\jmath}\\
|\overrightarrow{BD}|=\sqrt{(-2)^2+5^2}=\sqrt{4+25}=\sqrt{29}
\end{array}$$

ZADATAK 7.5.13

Točke $A(3,2)$, $B(1,-2)$ i $D(5,1)$ tri su vrha paralelograma $ABCD$.

Kolika je duljina dijagonale $=AC$?

RJEŠENJE

$$\frac{A(3,2),\quad B(1,-2),\quad D(5,1),\quad C(x_C,y_C)}{|\overline{AC}|=?}$$

$$\overrightarrow{AB}=\overrightarrow{DC}$$

$$(1-3)\vec{\imath}+(-2-2)\vec{\jmath}=(x_C-5)\vec{\imath}+(y_C-1)\vec{\jmath}$$

$$-2\vec{\imath}-4\vec{\jmath}=(x_C-5)\vec{\imath}+(y_C-1)\vec{\jmath}$$

$$x_C-5=-2\implies x_C=3$$

$$y_C-1=-4\implies y_C=-3$$

$$\implies C(3,-3)$$

$$\overrightarrow{AC}=(3-3)\vec{\imath}+(-3-2)\vec{\jmath}=-5\vec{\jmath}$$

$$|\overrightarrow{AC}|=\sqrt{0+5^2}=\sqrt{25}=5$$

ZADATAK 7.5.16

Odredi jedinični vektor istog smjera i orijentacije kao i vektor $\overrightarrow{AB}$, $A(3,1)$, $B(-1,-2)$.

RJEŠENJE

$$\frac{A(3,1),\quad B(-1,-2)}{\overrightarrow{AB}=(-1-3)\vec{\imath}+(-2-1)\vec{\jmath}=-4\vec{\imath}-3\vec{\jmath}}$$

$$|\overrightarrow{AB}|=\sqrt{(-4)^2+(-3)^2}=\sqrt{16+9}=5$$

$$\vec{e}=\frac{1}{|\overrightarrow{AB}|}\cdot\overrightarrow{AB}=\frac{1}{5}(-4\vec{\imath}-3\vec{\jmath})$$

$$\vec{e}=-\frac{4}{5}\vec{\imath}-\frac{3}{5}\vec{\jmath}$$

ZADATAK 7.5.17

Odredi jedinični vektor koji ima isti smjer, ali suprotnu orijentaciju od vektora $\overrightarrow{AB}$, $A(-4,9)$, $B(-2,5)$.

RJEŠENJE

$$\frac{A(-4,9),\quad B(-2,5)}{\overrightarrow{AB}=(-2+4)\vec{\imath}+(5-1)\vec{\jmath}=2\vec{\imath}-4\vec{\jmath}}$$

$$|\overrightarrow{AB}|=\sqrt{2^2+(-4)^2}=\sqrt{4+16}=\sqrt{20}=2\sqrt{5}$$

$$\vec{e}=\frac{1}{|\overrightarrow{AB}|}\cdot\overrightarrow{AB}=\frac{1}{2\sqrt{5}}(2\vec{\imath}-4\vec{\jmath})$$

$$\vec{e}=\frac{1}{\sqrt{5}}\vec{\imath}-\frac{2}{\sqrt{5}}\vec{\jmath}$$

ZADATAK 7.5.18

Odredi vektor $\vec{v}$ kolinearan s vektorom $\overrightarrow{AB}$, gdje je $A(2,-1)$, $B(-1,3)$ ako je $|\vec{v}|=20$.

RJEŠENJE

$$\begin{array}{l} \vec{v}=k\overrightarrow{AB}, \\ |\vec{v}|=20, \\ A(2,-1),\ B(-1,3) \\ \hline \overrightarrow{AB}=(2-1)\vec{\imath}+(3+1)\vec{\jmath}=-3\vec{\imath}+4\vec{\jmath} \\ |\overrightarrow{AB}|=\sqrt{9+16}=5 \\ \vec{e}=\dfrac{1}{|\overrightarrow{AB}|} \\ \overrightarrow{AB}=\dfrac{1}{5}(-3\vec{\imath}+4\vec{\jmath})=-\dfrac{3}{5}\vec{\imath}+\dfrac{4}{5}\vec{\jmath} \\ \vec{v}=\pm|\vec{v}|\cdot\vec{e}=\pm 20\left(-\dfrac{3}{5}\vec{\imath}+\dfrac{4}{5}\vec{\jmath}\right) \\ \vec{v_1}=-12\vec{\imath}+16\vec{\jmath} \\ \vec{v_2}=12\vec{\imath}-16\vec{\jmath} \end{array}$$

ZADATAK 7.5.20

Točke $A(1,-1)$, $B(3,3)$ i $C(4,5)$ pripadaju jednom pravcu. Provjeri!

RJEŠENJE

Ako su točke $A(-1,1)$, $B(3,3)$ i $C(4,5)$ na istom pravcu, tada mora biti:

$$\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}.$$

$$\begin{gathered} \overrightarrow{AC}=(4-1)\vec{\imath}+(5-(-1))\vec{\jmath}=3\vec{\imath}+6\vec{\jmath} \\ \overrightarrow{AB}=(3-1)\vec{\imath}+(3-(-1))\vec{\jmath}=2\vec{\imath}+4\vec{\jmath} \\ \overrightarrow{BC}=(4-3)\vec{\imath}+(5-3)\vec{\jmath}=\vec{\imath}+2\vec{\jmath} \\ \overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC} \\ 3\vec{\imath}+6\vec{\jmath}=2\vec{\imath}+4\vec{\jmath}+\vec{\imath}+2\vec{\jmath} \\ 3\vec{\imath}+6\vec{\jmath}=3\vec{\imath}+6\vec{\jmath} \end{gathered}$$

ZADATAK 7.5.24

Odredi nepoznatu koordinatu točke $B(x,2)$ tako da duljina vektora $\overrightarrow{AB}$, $A(-3,1)$ bude jednaka $5\sqrt{2}$.

RJEŠENJE

$$\begin{array}{l} |AB|=\sqrt{(x+3)^2+1} \\ 5\sqrt{2}=\sqrt{(x+3)^2+1} \quad /^2 \\ 50=x^2+6x+9+1 \\ x^2+6x-40=0 \\ x_{1,2}=\dfrac{-6\pm\sqrt{36+160}}{2}=\dfrac{-6\pm 14}{2} \\ x_1=-10, \quad x_2=4 \end{array}$$

ZADATAK 7.6.1

Kut između vektora $\vec{a}$ i $\vec{b}$ je 120°. Ako je $|\vec{a}| = 5$, $|\vec{b}| = 4$, koliko je

1) $\vec{a} \cdot \vec{b}$

2) $(\vec{a} + \vec{b})^2$

3) $(\vec{a} - \vec{b})^2$

4) $(\vec{a} - 2\vec{b}) \cdot (2\vec{a} + \vec{b})$?

RJEŠENJE

1) $\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cos\angle(\vec{a}, \vec{b}) = 5 \cdot 4 \cdot \cos 120^\circ = 20 \cdot \left(\frac{1}{2}\right) = -10$

2) $(\vec{a} + \vec{b})^2 = \vec{a}^2 + 2\vec{a} \cdot \vec{b} + \vec{b}^2 = |\vec{a}|^2 + 2|\vec{a}| \cdot |\vec{b}| \cos\angle(\vec{a}, \vec{b}) + |\vec{b}|^2$

$$= 5^2 + 2 \cdot 5 \cdot 4 \cdot \left(-\frac{1}{2}\right) + 4^2 = 25 - 20 + 16 = 21$$

3) $(\vec{a} - \vec{b})^2 = \vec{a}^2 - 2\vec{a} \cdot \vec{b} + \vec{b}^2 = |\vec{a}|^2 - 2|\vec{a}| \cdot |\vec{b}| \cos\angle(\vec{a}, \vec{b}) + |\vec{b}|^2$

$$= 25 - 2 \cdot 5 \cdot 4 \cdot \left(-\frac{1}{2}\right) + 16 = 25 + 20 + 16 = 61$$

4) $(\vec{a} - 2\vec{b}) \cdot (2\vec{a} + \vec{b}) = 2\vec{a}^2 - 4\vec{a}\vec{b} + \vec{a}\vec{b} - 2\vec{b}^2 = 2|\vec{a}|^2 - 3\vec{a}\vec{b} - 2|\vec{b}|^2$.

$$= 50 - 3|\vec{a}||\vec{b}| \cos\angle(\vec{a}, \vec{b}) - 2 \cdot 16 = 18 - 3 \cdot 5 \cdot 4 \cdot \left(-\frac{1}{2}\right) = 18 + 30 = 48$$

ZADATAK 7.6.2

Izračunaj umnožak $(3\vec{a} - 2\vec{b}) \cdot (\vec{a} + \vec{b})$ ako je $|\vec{a}| = 2$, $|\vec{b}| = 3$ te ako je kut između $\vec{a}$ i $\vec{b}$ 120°.

RJEŠENJE

$$\begin{aligned}
(3\vec{a} - 2\vec{b})(\vec{a} + \vec{b}) &= 3\vec{a}^2 + 3\vec{a}\vec{b} - 2\vec{a}\vec{b} - 2\vec{b}^2 \\
&= 3|\vec{a}|^2 + \vec{a}\vec{b} - 2|\vec{b}|^2 \\
&= 3|\vec{a}|^2 + |\vec{a}| \cdot |\vec{b}| \cos\angle(\vec{a}, \vec{b}) - 2|\vec{b}|^2 \\
&= 3 \cdot 4 + 2 \cdot 3 \cdot \left(-\frac{1}{2}\right) - 2 \cdot 9 \\
&= 12 - 3 - 18 = -9.
\end{aligned}$$

ZADATAK 7.6.4
Neka su $\vec{e}_1$ i $\vec{e}_2$ jedinični vektori i neka je $|\vec{e}_1+\vec{e}_2|=\sqrt{3}$.

Koliko je $(\vec{e}_1-2\vec{e}_2)\cdot(3\vec{e}_1+\vec{e}_2)$?

RJEŠENJE

$$\begin{aligned}
|\vec{e}_1+\vec{e}_2|^2 &= (\vec{e}_1-\vec{e}_2)^2\\
&= \vec{e}_1^{\,2}+2\vec{e}_1\vec{e}_2+\vec{e}_2^{\,2}\\
&= |\vec{e}_1|^2+2|\vec{e}_1||\vec{e}_2|\cos\angle(\vec{e}_1,\vec{e}_2)+|\vec{e}_2|^2\\
&= 1+2\cdot1\cdot1\cdot\cos\angle(\vec{e}_1,\vec{e}_2)+1^1\\
&= 2+2\cos\angle(\vec{e}_1,\vec{e}_2)\\
|\vec{e}_1+\vec{e}_2|^2 &= \sqrt{3}^2\\
3 &= 2+2\cos\angle(\vec{e}_1,\vec{e}_2)\\
1 &= 2\cos\angle(\vec{e}_1,\vec{e}_2)/:2\\
\frac{1}{2} &= \cos\angle(\vec{e}_1,\vec{e}_2)\\
60^\circ &= \angle(\vec{e}_1,\vec{e}_2)\\
(\vec{e}_1-2\vec{e}_2)(3\vec{e}_1+\vec{e}_2) &= 3\vec{e}_1^{\,2}+\vec{e}_1\cdot\vec{e}_2-6\vec{e}_1\cdot\vec{e}_2-2\vec{e}_2^{\,2}\\
&= 3\vec{e}_1^{\,2}-5\vec{e}_1\cdot\vec{e}_2-2\vec{e}_2^{\,2}\\
&= 3|\vec{e}_1|^2-5|\vec{e}_1|\cdot|\vec{e}_2|\cos\angle(\vec{e}_1,\vec{e}_2)-2|\vec{e}_2|^2\\
&= 3\cdot1-5\cdot1\cdot1\cdot\frac{1}{2}-2\cdot1\\
&= 3-\frac{5}{2}-2=-\frac{3}{2}
\end{aligned}$$

ZADATAK 7.6.5
Odredi vektor $\vec{b}$ kolinearan s vektorom $\vec{a}=\vec{\imath}-2\vec{\jmath}$ ako je $\vec{a}\cdot\vec{b}=-15$.

RJEŠENJE
$\vec{b}$ je kolinearan s $\vec{a}\implies\vec{b}\parallel\vec{a}\implies\angle(\vec{a},\vec{b})=180^\circ$.

$$\begin{aligned}
\vec{b} &= b_x\vec{\imath}+b_y\vec{\jmath}\\
\vec{a}\cdot\vec{b} &= a_xb_x+a_yb_y\\
&= 1\cdot b_x+(-2)b_y\\
b_x-2b_y &= -15\\
b_x &= 2b_y-15
\end{aligned}$$

$|\vec{a}|=\sqrt{1+4}=\sqrt{5}$, $\vec{b}=\lambda\vec{a}$ $\cos\angle(\vec{a},\vec{b})=-1=\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}$

$$\begin{aligned}
-1 &= \frac{-15}{\sqrt{5}\cdot|\lambda|\sqrt{5}}\\
-1 &= \frac{-15}{5|\lambda|}/\cdot5|\lambda|\\
-5|\lambda| &= -15\\
|\lambda| &= 3
\end{aligned}$$

$\implies\lambda=-3$ jer je $\angle(\vec{a},\vec{b})=180^\circ$.

$\vec{b}=-3(\vec{\imath}-2\vec{\jmath})=-3\vec{\imath}+6\vec{\jmath}$.

ZADATAK 7.6.9

Odredi kut između vektora $\vec{p}+\vec{q}$ i $\vec{p}-\vec{q}$ ako je $\vec{p}=3\vec{\imath}-2\vec{\jmath}$, $\vec{q}=-\vec{\imath}+4\vec{\jmath}$.

RJEŠENJE

$\vec{p}+\vec{q}=3\vec{\imath}-2\vec{\jmath}-\vec{\imath}+4\vec{\jmath}=2\vec{\imath}+2\vec{\jmath}$,

$\vec{p}-\vec{q}=3\vec{\imath}-2\vec{\jmath}+\vec{\imath}-4\vec{\jmath}=4\vec{\imath}-6\vec{\jmath}$

$$\begin{aligned}\cos\angle(\vec{p}+\vec{q},\vec{p}-\vec{q}) &= \frac{(\vec{p}+\vec{q})\cdot(\vec{p}-\vec{q})}{|\vec{p}+\vec{q}|\cdot|\vec{p}-\vec{q}|}=\frac{2\cdot4+2\cdot(-6)}{\sqrt{4+4}\cdot\sqrt{16+36}}\\ &= \frac{8-12}{2\sqrt{2}\cdot\sqrt{52}}=\frac{-4}{4\sqrt{26}}=-\frac{1}{\sqrt{26}}\end{aligned}$$

$\angle(\vec{p}+\vec{q},\vec{p}-\vec{q})\approx 101^\circ18'$.

ZADATAK 7.6.10

Ako su dani vektori $\vec{a}=3\vec{\imath}+4\vec{\jmath}$ i $\vec{b}=-5\vec{\imath}+2\vec{\jmath}$, koliki kut zatvaraju vektori $\vec{a}+\vec{b}$ i $\vec{a}-\vec{b}$?

RJEŠENJE

$$\begin{aligned}&\vec{a}=3\vec{\imath}+4\vec{\jmath}\\ &\underline{\vec{b}=-5\vec{\imath}+2\vec{\jmath}}\\ &\vec{a}+\vec{b}=-2\vec{\imath}+6\vec{\jmath}\\ &\vec{a}-\vec{b}=8\vec{\imath}+2\vec{\jmath}\\ &\cos\varphi=\frac{(-2)\cdot8+2\cdot6}{\sqrt{4+36}\cdot\sqrt{64+4}}=\frac{-4}{\sqrt{2720}}=-0.0766965\\ &\varphi=94^\circ24'\end{aligned}$$

ZADATAK 7.6.11

Ako je $\vec{a}=-\vec{\imath}+2\vec{\jmath}$, $\vec{b}=3\vec{\imath}+4\vec{\jmath}$, koliki kut zatvaraju vektori $\vec{a}+\vec{b}$ i $\vec{a}-\vec{b}$?

RJEŠENJE

$\vec{a}+\vec{b}=-\vec{\imath}+2\vec{\jmath}+3\vec{\imath}+4\vec{\jmath}=2\vec{\imath}+6\vec{\jmath}$

$\vec{a}-\vec{b}=-\vec{\imath}+2\vec{\jmath}-3\vec{\imath}-4\vec{\jmath}=-4\vec{\imath}-2\vec{\jmath}$

$$\begin{aligned}\cos\angle(\vec{a}+\vec{b},\vec{a}-\vec{b}) &= \frac{(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})}{|\vec{a}+\vec{b}|\cdot|\vec{a}-\vec{b}|}=\frac{2\cdot(-4)+6\cdot(-2)}{\sqrt{4+36}\cdot\sqrt{16+4}}\\ &= \frac{-8-12}{2\sqrt{10}\cdot2\sqrt{5}}=\frac{-5}{\sqrt{50}}=-\frac{5}{5\sqrt{2}}\\ &= -\frac{1}{\sqrt{2}}=-\frac{\sqrt{2}}{2}\end{aligned}$$

$\angle(\vec{a}+\vec{b},\vec{a}-\vec{b})=135^\circ$.

ZADATAK 7.6.13

Kolika je duljina vektora $\vec{v} = 3\vec{a} + 2\vec{b}$ ako je $|\vec{a}| = 2$, $|\vec{b}| = \sqrt{2}$ te $\angle(\vec{a}, \vec{b}) = \dfrac{3\pi}{4}$?

RJEŠENJE

$$\begin{aligned}
|\vec{v}|^2 &= (3\vec{a}2\vec{b})^2 = 9\vec{a}^2 + 12\vec{a}\cdot\vec{b} + 4\vec{b}^2 \\
&= 9|\vec{a}|^2 + 12|\vec{a}|\cdot|\vec{b}|\cos\frac{3\pi}{4} + 4|\vec{b}|^2 \\
&= 9\cdot 4 + 12\cdot 2\cdot\sqrt{2}\cdot\left(-\frac{\sqrt{2}}{2}\right) + 4\cdot\sqrt{2}^2 \\
&= 36 - 24 + 8 = 20 \\
|\vec{v}| &= \sqrt{20} = 2\sqrt{5}
\end{aligned}$$

ZADATAK 7.6.14

ISPITNI Odredi vektor $\vec{b}$ okomit na vektor $\vec{a} = -2\vec{\imath} + \vec{\jmath}$ ako je $|\vec{b}| = \sqrt{5}$.

RJEŠENJE

Iz

$$\vec{b}\perp\vec{a} \implies \vec{a}\cdot\vec{b} = 0 \implies -2b_x + b_y = 0 \implies b_y = 2b_x \quad .$$

Iz

$$|\vec{b}| = \sqrt{5} \implies \sqrt{b_x^2 + b_y^2} = \sqrt{5}/^2 \implies b_x^2 + 4b_x^2 = 5$$

$$\implies b_x^2 = 1 \implies b_x = \pm 1 \implies b_y = \pm 2$$

$\vec{b} = \pm(\vec{\imath} + 2\vec{\jmath})$.

ZADATAK 7.6.16

Ako su dane točke $A(1,4)$, $B(3,3)$, $C(-5,3)$, $D(1,5)$, odredi kut između vektora $\overrightarrow{AB}$ i $\overrightarrow{CD}$.

RJEŠENJE

$$\begin{aligned}
\overrightarrow{AB} &= (3-1)\vec{\imath} + (3-4)\vec{\jmath} = 2\vec{\imath} - \vec{\jmath} \\
\overrightarrow{CD} &= (1+5)\vec{\imath} + (5-3)\vec{\jmath} = 6\vec{\imath} + 2\vec{\jmath} \\
\cos\varphi &= \frac{\overrightarrow{AB}\cdot\overrightarrow{CD}}{|\overrightarrow{AB}|\cdot|\overrightarrow{CD}|} = \frac{2\cdot 6 - 1\cdot 2}{\sqrt{4+1}\cdot\sqrt{36+4}} = \frac{10}{\sqrt{5}\cdot 2\sqrt{10}} \\
&= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \\
\varphi &= 45^\circ
\end{aligned}$$

ZADATAK 7.6.18

ISPITNI Koliki su kutovi trokuta ABC ako su vrhovi trokuta točke $A(-1,3)$, $B(1,-1)$, $C(2,4)$?

RJEŠENJE

$$\underline{A(-1,3), \quad B(1,-1), \quad C(2,4) \qquad\qquad}$$

$$\overrightarrow{AB} = 2\vec{\imath} - 4\vec{\jmath}, \quad |\overrightarrow{AB}| = \sqrt{4+16} = 2\sqrt{5}$$

$$\overrightarrow{AC} = 3\vec{\imath} + \vec{\jmath}, \quad |\overrightarrow{AC}| = \sqrt{9+1} = \sqrt{10}$$

$$\cos\alpha = \frac{\overrightarrow{AB}\cdot\overrightarrow{AC}}{|\overrightarrow{AB}|\cdot|\overrightarrow{AC}|} = \frac{2\cdot 3 + (-4)\cdot 1}{2\sqrt{5}\cdot\sqrt{10}} = \frac{2}{10\sqrt{2}} = \frac{\sqrt{2}}{10}$$

$$\alpha = 81^\circ 52'$$

$$\overrightarrow{BA} = -2\vec{\imath} + 4\vec{\jmath}, \quad |\overrightarrow{BA}| = \sqrt{4+16} = 2\sqrt{5}$$

$$\overrightarrow{BC} = \vec{\imath} + 5\vec{\jmath}, \quad |\overrightarrow{BC}| = \sqrt{1+25} = \sqrt{26}$$

$$\cos\beta = \frac{\overrightarrow{BA}\cdot\overrightarrow{BC}}{|\overrightarrow{BA}|\cdot|\overrightarrow{BC}|} = \frac{(-2)\cdot 1 + 4\cdot 5}{2\sqrt{5}\cdot\sqrt{26}} = \frac{18}{\sqrt{520}} = 0.78935$$

$$\beta = 37^\circ 52'$$

$$\gamma = 180^\circ - \alpha - \beta = 60^\circ 16'$$

ZADATAK 7.6.22

Odredi najveći kut trokuta ABC ako je $A(-1,3)$, $B(1,1)$, $C(5,3)$.

RJEŠENJE

$$\overrightarrow{AB} = (1+1)\vec{\imath} + (1-3)\vec{\jmath} = 2\vec{\imath} - 2\vec{\jmath}$$

$$\overrightarrow{BA} = -2\vec{\imath} + 2\vec{\jmath}$$

$$\overrightarrow{AC} = (5+1)\vec{\imath} + (3-3)\vec{\jmath} = 6\vec{\imath}$$

$$\overrightarrow{BC} = (5-1)\vec{\imath} + (3-1)\vec{\jmath} = 4\vec{\imath} + 2\vec{\jmath}$$

$$|\overrightarrow{AB}| = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$|\overrightarrow{AC}| = \sqrt{36+0} = 6$$

$$|\overrightarrow{BC}| = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

$$\cos\beta = \frac{\overrightarrow{BA}\cdot\overrightarrow{BC}}{|\overrightarrow{BA}|\cdot|\overrightarrow{BC}|} = \frac{-2\cdot 4 + 2\cdot 2}{2\sqrt{2}\cdot 2\sqrt{5}} = \frac{-4}{4\sqrt{10}} = -\frac{1}{\sqrt{10}}$$

$$\beta = 108^\circ 26'$$

ZADATAK 7.6.23

Odredi kut između dijagonala paralelograma $ABCD$ ako je $\overrightarrow{AB}=4\vec{\imath}-3\vec{\jmath}$, $\overrightarrow{AD}=6\vec{\imath}+\vec{\jmath}$.

RJEŠENJE

$\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BD}=4\vec{\imath}-3\vec{\jmath}+6\vec{\imath}+\vec{\jmath}=10\vec{\imath}-2\vec{\jmath}$,

$|\overrightarrow{AC}|=\sqrt{100+4}=\sqrt{104}=2\sqrt{26}$

$\overrightarrow{BD}=\overrightarrow{AD}-\overrightarrow{AB}=6\vec{\imath}+\vec{\jmath}-4\vec{\imath}+3\vec{\jmath}=2\vec{\imath}+4\vec{\jmath}$,

$|\overrightarrow{BD}|=\sqrt{4+16}=\sqrt{20}=2\sqrt{5}$

$$\cos\varphi=\frac{\overrightarrow{AC}\cdot\overrightarrow{BD}}{|\overrightarrow{AC}|\cdot|\overrightarrow{BD}|}=\frac{10\cdot 2-2\cdot 4}{2\sqrt{26}\cdot 2\sqrt{5}}=\frac{12}{4\sqrt{130}}=\frac{3}{130}$$

$\varphi=\operatorname{arc}\cos\frac{3}{\sqrt{130}}\approx 74^\circ 44'$.

ZADATAK 7.6.28

Ako su $\vec{a}$ i $\vec{b}$ vektori za koje je $|\vec{a}|=11$, $|\vec{b}|=23$, $|\vec{a}-\vec{b}|=30$, koliko je $|\vec{a}+\vec{b}|$?

RJEŠENJE

$|\vec{a}+\vec{b}|^2+|\vec{a}-\vec{b}|^2=(\vec{a}+\vec{b})^2+(\vec{a}-\vec{b})^2=\vec{a}^2+2\vec{a}\vec{b}+\vec{b}^2+\vec{a}^2-2\vec{a}\vec{b}+\vec{b}^2=2(\vec{a}^2+\vec{b}^2)=2(|\vec{a}|^2+|\vec{b}|^2)$

$$\begin{aligned}|\vec{a}+\vec{b}|^2+30^2&=2(11^2+23^2)\\|\vec{a}+\vec{b}|^2&=2(121+529)-900\\|\vec{a}+\vec{b}|^2&=400\\|\vec{a}+\vec{b}|&=20\end{aligned}$$

ZADATAK 7.6.29

Ako je $|\vec{a}|=5$, $|\vec{a}+\vec{b}|=13$, $|\vec{a}-\vec{b}|=9$, kolika je duljina vektora $\vec{b}$?

RJEŠENJE

$$\begin{aligned}|\vec{a}+\vec{b}|^2&=(\vec{a}+\vec{b})^2=\vec{a}^2+2\vec{a}\vec{b}+\vec{b}^2=169,\\|\vec{a}-\vec{b}|^2&=(\vec{a}-\vec{b})^2=\vec{a}^2-2\vec{a}\vec{b}+\vec{b}^2=81\\2\vec{a}^2+2\vec{b}^2&=250\\2\cdot 5^2+2\vec{b}^2&=250\\2\vec{b}^2&=200\\\vec{b}^2&=100\\|\vec{b}|^2&=100\\|\vec{b}|&=10\end{aligned}$$

ZADATAK 7.6.41
Trokut s vrhovima $A(-3,4)$, $B(1,2)$, $C(-2,6)$ je pravokutan. Primjenom skalarnog umnoška provjeri ovu tvrdnju.

RJEŠENJE

$$\begin{aligned} &\underline{A(-3,4), \quad B(1,2), \quad C(-2,6) \qquad\qquad\qquad} \\ &\overrightarrow{AB} = 4\vec{\imath} - 2\vec{\jmath} \\ &\overrightarrow{AC} = \vec{\imath} + 2\vec{\jmath} \\ &\overrightarrow{AB} \cdot \overrightarrow{AC} = 4 \cdot 1 + (-2) \cdot 2 = 0 \\ &\implies \text{trokut je pravokutan.} \end{aligned}$$

ZADATAK 7.5.3
Točke $A(3,1)$, $B(0,5)$, $C(-2,-1)$ vrhovi su trokuta.

Dokaži da vrijedi: $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$.

RJEŠENJE
$\overrightarrow{AB} = -3\vec{\imath} + 4\vec{\jmath}$

$\overrightarrow{BC} = -2\vec{\imath} - 6\vec{\jmath}$

$\overrightarrow{CA} = 5\vec{\imath} + 2\vec{\jmath}$

$$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = -3\vec{\imath} + 4\vec{\jmath} - 2\vec{\imath} - 6\vec{\jmath} + 5\vec{\imath} + 2\vec{\jmath} = \vec{0}.$$

ZADATAK 7.5.5
Izračunaj duljine vektora iz zadatka 7.5.1.

RJEŠENJE

$\overrightarrow{AB} = -2\vec{\imath} - 4\vec{\jmath}$

$|\overrightarrow{AB}| = \sqrt{(-2)^2 + (-4)^2} = \sqrt{4+16} = 2\sqrt{5}$

$\overrightarrow{AC} = 5\vec{\imath} - 4\vec{\jmath}$

$|\overrightarrow{AC}| = \sqrt{5^2 + (-4)^2} = \sqrt{25+16} = \sqrt{41}$

$\overrightarrow{AD} = 7\vec{\imath} - 2\vec{\jmath}$

$|\overrightarrow{AD}| = \sqrt{7^2 + (-2)^2} = \sqrt{49+4} = \sqrt{53}$

$\overrightarrow{AE} = 5\vec{\imath} + 3\vec{\jmath}$

$|\overrightarrow{AE}| = \sqrt{5^2 + 3^2} = \sqrt{25+9} = \sqrt{34}$

$\overrightarrow{BC} = 7\vec{\imath}$

$|\overrightarrow{BC}| = \sqrt{7^2 + 0^2} = \sqrt{49} = 7$

$\overrightarrow{BD} = 9\vec{\imath} + 2\vec{\jmath}$

$|\overrightarrow{BD}| = \sqrt{9^2 + 2^2} = \sqrt{81+4} = \sqrt{85}$

$\overrightarrow{BE} = 7\vec{\imath} + 7\vec{\jmath}$

$|\overrightarrow{BE}| = \sqrt{7^2 + 7^2} = \sqrt{2 \cdot 49} = 7\sqrt{2}$

$\overrightarrow{CD} = 2\vec{\imath} + 2\vec{\jmath}$

$|\overrightarrow{CD}| = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$

$\overrightarrow{CE} = 7\vec{\jmath}$

$|\overrightarrow{CE}| = \sqrt{0^2 + 7^2} = \sqrt{49} = 7$

$\overrightarrow{DE} = -2\vec{\imath} + 5\vec{\jmath}$

$|\overrightarrow{DE}| = \sqrt{(-2)^2 + 5^2} = \sqrt{4+25} = \sqrt{29}.$

ZADATAK 7.5.6
Dane su točke $A(-3,4), B(2,-1), C(3,0)$.

Odredi vektore $\overrightarrow{AB}, \overrightarrow{BC}$ i $\overrightarrow{CA}$ te provjeri da je $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{0}$.

RJEŠENJE

$A(-3,4),\ B(2,-1),\ C(3,0)$

$\overrightarrow{AB} = 5\vec{\imath} - 5\vec{\jmath}$

$\overrightarrow{BC} = \vec{\imath} + \vec{\jmath}$

$\overrightarrow{CA} = -6\vec{\imath} + 4\vec{\jmath}$

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 5\vec{\imath} - 5\vec{\jmath} + \vec{\imath} + \vec{\jmath} - 6\vec{\imath} + 4\vec{\jmath} = \overrightarrow{0}$.

ZADATAK 7.5.8
Ako su $A(1,-1)$, $B(3,2)$ i $C(-2,3)$ tri uzastopna vrha paralelograma $ABCD$, odredi koordinate četvrtog vrha D.

RJEŠENJE

$$\begin{array}{l}
\underline{A(1,-1), \quad B(3,2), \quad C(-2,3), \quad D(x,y)} \\
\overrightarrow{AB}=\overrightarrow{DC} \quad \text{(paralelogram)} \\
(3-1)\vec{\imath}+(2+1)\vec{\jmath}=(-2-x)\vec{\imath}+(3-y)\vec{\jmath} \\
2\vec{\imath}+3\vec{\jmath}=(-2-x)\vec{\imath}+(3-y)\vec{\jmath} \\
2=-2-x \implies x=-4 \\
3=3-y \implies y=0 \\
\implies D(-4,0)
\end{array}$$

ZADATAK 7.5.14
Točke $A(-2,1)$, $B(3,0)$, $C(2,5)$, $D(-3,6)$ su vrhovi romba. Dokaži!

RJEŠENJE

$$\begin{array}{lr}
\overrightarrow{AB}=\overrightarrow{DC} & (1) \\
\overrightarrow{AD}=\overrightarrow{BC} & (2) \\
|\overrightarrow{AB}|=|\overrightarrow{DC}|=|\overrightarrow{AD}|=|\overrightarrow{BC}| & (3) \\
\hline
\end{array}$$

$$(1)\begin{cases}\overrightarrow{AB}=(3+2)\vec{\imath}+(0-1)\vec{\jmath}=5\vec{\imath}-\vec{\jmath} \\ \overrightarrow{DC}=(2+3)\vec{\imath}+(5-6)\vec{\jmath}=5\vec{\imath}-\vec{\jmath}\end{cases}$$

$$(2)\begin{cases}\overrightarrow{AD}=(-3+2)\vec{\imath}+(6-1)\vec{\jmath}=-\vec{\imath}+5\vec{\jmath} \\ \overrightarrow{BC}=(2-3)\vec{\imath}+(5-0)\vec{\jmath}=-\vec{\imath}+5\vec{\jmath}\end{cases}$$

$$(3)\begin{cases}|\overrightarrow{AB}|=\sqrt{5^2+(-1)^2}=\sqrt{26}=|\overrightarrow{DC}| \\ |\overrightarrow{AD}|=\sqrt{(-1)^2+5^2}=\sqrt{26}=|\overrightarrow{BC}|\end{cases}$$

ZADATAK 7.5.15

Točke $A(1,3)$, $B(7,1)$, $D(3,9)$ tri su vrha paralelograma.

1) Dokaži da je taj paralelogram kvadrat.

2) Kolika je površina kvadrata?

3) Kolike su duljine dijagonala kvadrata?

RJEŠENJE

1)

$$\begin{aligned}
&\underline{A(1,3), \quad B(7,1), \quad D(3,9)} \\
&\overrightarrow{AB} = (7-1)\vec{\imath} + (1-3)\vec{\jmath} = 6\vec{\imath} - 2\vec{\jmath} \\
&\overrightarrow{AD} = (3-1)\vec{\imath} + (9-3)\vec{\jmath} = 2\vec{\imath} + 6\vec{\jmath} \\
&|\overrightarrow{AB}| = \sqrt{6^2 + (-2)^2} = \sqrt{40} \\
&|\overrightarrow{AD}| = \sqrt{2^2 + 6^2} = \sqrt{40} \\
&a = |\overrightarrow{AB}| = |\overrightarrow{AD}| = \sqrt{40}
\end{aligned}$$

2) $P = a^2 = 40$

3) $d = |\overrightarrow{BD}| = \sqrt{4^2 + 8^2} = \sqrt{48} = 4\sqrt{3}$

$\overrightarrow{BD} = (3-7)\vec{\imath} + (9-1)\vec{\jmath} = 4\vec{\imath} + 8\vec{\jmath}$.

ZADATAK 7.5.19
Točke $A(2,1)$, $B(3,4)$ i $C(5,10)$ su kolinearne, pripadaju jednom pravcu. Provjeri ovu tvrdnju!

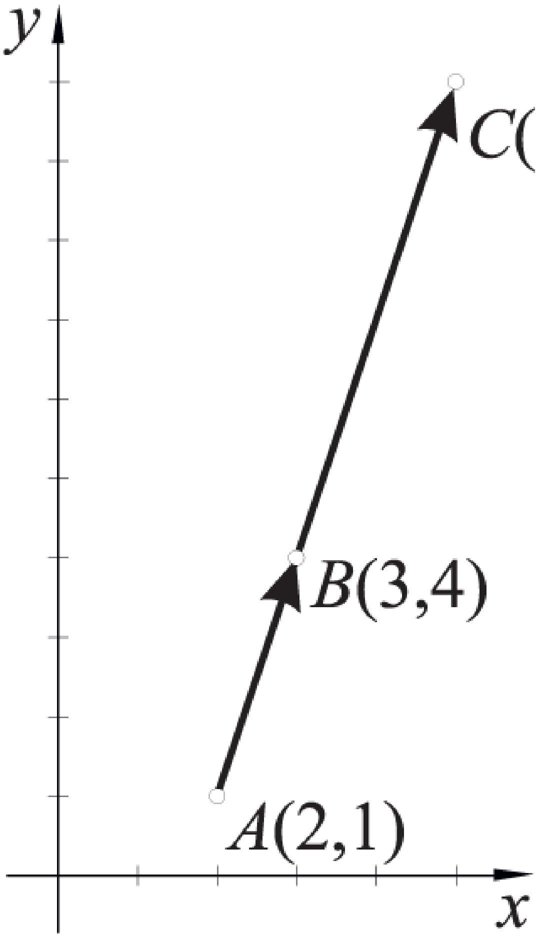
RJEŠENJE
I. način: $A(2,1)$, $B(3,4)$, $C(5,10)$ Ako su tri točke kolinearne, tada mora biti:

$$x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0$$

Uvrstimo vrijednosti:

$$\begin{gathered}2\cdot(4-10)+3\cdot(10-1)+5\cdot(1-4)=0\\ 2\cdot(-6)+3\cdot 9+5\cdot(-3)=0\\ -12+27-15=0\\ 0=0\end{gathered}$$

II. način: skiciramo položaj tih točaka i sa slike vidimo da mora biti $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}$



$$\begin{gathered}\overrightarrow{AC}=(5-2)\vec{\imath}+(10-1)\vec{\jmath}=3\vec{\imath}+9\vec{\imath}\\ \overrightarrow{AB}=(3-2)\vec{\imath}+(4-1)\vec{\jmath}=\vec{\imath}+3\vec{\imath}\\ \overrightarrow{BC}=(5-3)\vec{\imath}+(10-4)\vec{\jmath}=2\vec{\imath}+6\vec{\imath}\\ \overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}\\ 3\vec{\imath}+9\vec{\jmath}=\vec{\imath}+3\vec{\jmath}+2\vec{\imath}+6\vec{\jmath}\\ 3\vec{\imath}+9\vec{\jmath}=3\vec{\imath}+9\vec{\jmath}\end{gathered}$$

ZADATAK 7.5.21
Točke $A(-3,0)$, $B(0,2)$ i $C(6,6)$ pripadaju jednom pravcu. Provjeri!

RJEŠENJE
Odredimo $\overrightarrow{AB}=3\vec{\imath}+2\vec{\jmath}$, $\overrightarrow{BC}=6\vec{\imath}+4\vec{\jmath}$. Očito je $\overrightarrow{BC}=2\cdot\overrightarrow{AB}$, a to znači da su vektori $\overrightarrow{AB}$ i $\overrightarrow{BC}$ kolinearni, odnosno da su točke A, B i C na jednom pravcu.